Fuzzy p-ideal in INK-Algebra

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Abstract - In this paper we present the perceptions of fuzzy p-ideals in INK-algebras and explore nearly of its chattels. We define the upper level subset $\mathcal{I}$ of $\mathcal{I}$ and the cartesian product of $\mathcal{I}$ and $\mathcal{I}$ from $\mathcal{I} \times \mathcal{I}$ to $[0, 1]$, for all elements $(\sigma, \tau)$ in $\mathcal{I} \times \mathcal{I}$. We verified any subalgebra of an INK-algebra $\mathcal{I}$ can be comprehended as a level subalgebra of some F-subalgebra of $\mathcal{I}$ and $\mathcal{I}$ is a p-ideal of $\mathcal{I}$. Also, we proved, the cartesian product of $\mathcal{I}$ and $\mathcal{I}$ is a fuzzy p-ideal of $\mathcal{I} \times \mathcal{I}$.

Keywords – INK-algebra, Fuzzy subalgebra, Fuzzy ideal, Fuzzy P-ideal, homomorphism, Cartesian product, level subset.

I. INTRODUCTION

Isaki and Tanaka (1966) introduced two classes of abstract algebras BCI-algebras and BCK-algebras. It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebra. Hu and Li (1983) introduced a wide class of abstract algebra namely BCH- algebras. In (2017) Kaviyarasu and Indhira introduced new notion is called a INK-algebras. Zadeh (1965) introduced the notion of fuzzy sets. This concept has been applied to many mathematical branches. Xi applied this concept to BCK-algebra. Dudek and Jun (2001) fuzzified the ideals in BCC-algebras. In this study, we introduce the concepts of fuzzy subalgebras and fuzzy p-ideal in INK-algebra and investigate some of their properties.

II. PRELIMINARIES

2.1 Definition An algebra $\mathcal{I}$ is called a INK-algebra if it is satisfies the following conditions, for any $\sigma, \tau, \rho$ in $\mathcal{I}$:

INK-1: $(\sigma \circ \tau \circ (\sigma \circ \rho)) \circ (\rho \circ \tau) = 0$
INK-2: $((\sigma \circ \rho) \circ (\tau \circ \rho)) \circ (\sigma \circ \tau) = 0$
INK-3: $\sigma \circ 0 = \sigma$
INK-4: $\sigma \circ \tau = 0$ and $\tau \circ \sigma = 0$ imply $\sigma = \tau$.

We call the “$\circ$” is binary operation and the “0” is constant of $\mathcal{I}$.

2.2 Definition A non-void subset $S$ of an INK - algebra $(\mathcal{I}, \circ, 0)$ is said to be a subalgebra of $\mathcal{I}$, if $\sigma \circ \tau \in S$. whenever $\sigma, \tau \in S$.

2.3 Definition Let $(\mathcal{I}, \circ, 0)$ be a INK-algebra. A non- empty sub set $I$ of $\mathcal{I}$ is called a p- ideal of $\mathcal{I}$ if it satisfies,

$\bullet$ $0 \in I$
$\bullet (\sigma \circ \rho) \circ (\tau \circ \rho) \in I$ and $\tau \in I$ imply $\sigma \in I$, $\tau \in I$, for all $\sigma, \tau, \rho \in \mathcal{I}$.

2.4 Definition Let $\mathcal{I}$ be a non-empty set. A Fuzzy set can be defined as an object of the form

$\mathcal{I} = \{ (\mathcal{I}, 9 (\sigma)): \sigma \in \mathcal{I} \}$, where the function $9: \mathcal{I} \rightarrow [0, 1]$ is the degree of membership.
2.5 Definition. A fuzzy set \( \mathcal{F} \) in a INK-algebra \( \mathfrak{A} \) is called a FINK-subalgebra of \( \mathfrak{A} \) if
\[
\mathcal{F}(\sigma \odot \tau) \geq \min \{ \mathcal{F}(\sigma), \mathcal{F}(\tau) \}, \text{ for all } \sigma, \tau \in \mathfrak{A}.
\]

2.6 Definition. Let \( \mathcal{F} \) be a fuzzy set of a set \( \mathfrak{A} \). For a fixed \( t \in [0, 1] \), the set \( \mathcal{F}_t = \{ \sigma \in \mathfrak{A} / \mathcal{F}(\sigma) \geq t \} \) is called an upper level of \( \mathcal{F} \).

III. FUZZY P-IDEAL OF INK-ALGEBRA

3.1 Definition. A fuzzy set \( \mathcal{F} \) in a INK-algebra \( \mathfrak{A} \) is called a fuzzy ideal of \( \mathfrak{A} \), if:

- \( \mathcal{F}(0) \geq \mathcal{F}(\sigma) \)
- \( \mathcal{F}(\sigma) \geq \min \{ \mathcal{F}(\sigma \odot \tau), \mathcal{F}(\tau) \} \) for all \( \sigma, \tau \in \mathfrak{A} \).

3.2 Definition. A fuzzy set \( \mathcal{F} \) in a INK-algebra \( \mathfrak{A} \) is called a fuzzy p-ideal of \( \mathfrak{A} \), if:

- \( \mathcal{F}(0) \geq \mathcal{F}(\sigma) \)
- \( \mathcal{F}(\sigma) \geq \min \{ \mathcal{F}((\sigma \odot \tau) \odot (\tau \odot \rho)), \mathcal{F}(\tau) \} \) for all \( \sigma, \tau, \rho \in \mathfrak{A} \).

3.3 Lemma. If \( \mathcal{F} \) is a fuzzy INK-subalgebra of a INK-algebra \( \mathfrak{A} \), then \( \mathcal{F}(0) \geq \mathcal{F}(\sigma) \) for any \( \sigma \in \mathfrak{A} \).

Proof:
Since \( \sigma \odot \sigma = 0 \) for any \( \sigma \in \mathfrak{A} \), then:
\[
\mathcal{F}(0) = \mathcal{F}(\sigma \odot \sigma) \\
\geq \min \{ \mathcal{F}(\sigma), \mathcal{F}(\sigma) \} \\
= \mathcal{F}(\sigma).
\]

3.4 Theorem. A fuzzy set \( \mathcal{F} \) of a INK-algebra \( \mathfrak{A} \) is a fuzzy INK-subalgebra if and only if for every \( t \in [0, 1] \), \( \mathcal{F}_t \) is either empty or a subalgebra of \( \mathfrak{A} \).

Proof.
Assume that \( \mathcal{F} \) is a Fuzzy INK-subalgebra of \( \mathfrak{A} \) and \( \mathcal{F}_t \neq \emptyset \), for any \( \sigma, \tau \in \mathcal{F}_t \),
\[
\mathcal{F}(\sigma \odot \tau) \geq \min \{ \mathcal{F}(\sigma), \mathcal{F}(\tau) \} \geq t.
\]
Therefore \( \sigma \odot \tau \in \mathcal{F}_t \), \( \mathcal{F}_t \) is a subalgebra of \( \mathfrak{A} \).

Conversely, \( \mathcal{F}_t \) is a subalgebra of \( \mathfrak{A} \).

Let \( \sigma, \tau \in \mathfrak{A} \). Take \( t = \min \{ \mathcal{F}(\sigma), \mathcal{F}(\tau) \} \).

Then by assumption \( \mathcal{F}_t \) is a subalgebra of \( \mathfrak{A} \) implies:
\[
\sigma \odot \tau \in \mathcal{F}_t, \\
\mathcal{F}(\sigma \odot \tau) \geq t = \min \{ \mathcal{F}(\sigma), \mathcal{F}(\tau) \}.
\]

3.5 Theorem. Any subalgebra of a INK-algebra \( A \) can be realized as a level subalgebra of some Fuzzy INK-subalgebra of \( A \).

Proof:
Let \( \mathcal{F} \) be a subalgebra of a given INK-algebra \( A \) and let \( \mathcal{F} \) be a fuzzy set in \( A \) defined by:
\[
\mathcal{F}(\sigma) = \begin{cases} 
  t, & \text{if } \sigma \in A \\
  0, & \text{if } \sigma \notin A
\end{cases}
\]
where, \( t (0,1) \) is fixed. It is clear that \( \vartheta _1 = A \). Such defined \( \vartheta \) is a fuzzy subalgebra of \( A \).

Let \( \sigma , \tau \in A \). If \( \sigma , \tau \in A \) then also \( \sigma \odot \tau \in A \).

Hence \( \vartheta (\sigma ) = \vartheta (\tau ) = \vartheta (\sigma \odot \tau ) = t \) and

\[
\vartheta (\sigma \odot \tau ) \geq \min \{ \vartheta (\sigma ), \vartheta (\tau ) \}.
\]

If \( \sigma , \tau \not\in A \) then \( \vartheta (\sigma ) = \vartheta (\tau ) = 0 \) and in the consequence

\[
\vartheta (\sigma \odot \tau ) \geq \min \{ \vartheta (\sigma ), \vartheta (\tau ) \} = 0.
\]

If at most one of \( \sigma , \tau \) belongs to \( A \), then at least one of \( \vartheta (\sigma ) \) and \( \vartheta (\tau ) \) is equal to 0.

Therefore,

\[
\min \{ \vartheta (\sigma ), \vartheta (\tau ) \} = 0
\]

\[
\vartheta (\sigma \odot \tau ) \geq 0.
\]

3.6 Theorem. Two level subalgebras \( \vartheta _s, \vartheta _t (s < t) \) of a fuzzy INK-subalgebra are equal if and only if there is no \( \sigma \in \mathfrak{A} \) such that \( s \leq \vartheta (\sigma ) < t \).

\textbf{Proof:}

Let \( \vartheta _s = \vartheta _t \) for some \( s < t \), such that \( s \leq \vartheta (\sigma ) < t \), then \( \vartheta _t \subseteq \vartheta _s \),

which is a contradiction.

Conversely, assume that there is no \( \sigma \in \mathfrak{A} \),

such that \( s \leq \vartheta (\sigma ) < t \). If \( \sigma \in \mathfrak{A} \), then \( \vartheta (\sigma ) \geq s \) and \( \vartheta (\sigma ) \geq t \),

since \( \vartheta (\sigma ) \) does not lie between \( s \) and \( t \).

Thus, \( \vartheta _t \in \mathfrak{A} \), which gives \( \vartheta _s \subseteq \vartheta _t \). Also \( t \leq \vartheta _s \subseteq \vartheta _t \).

Therefore, \( \vartheta _s = \vartheta _t \).

3.7 Theorem. Every fuzzy \( P \)-ideal \( \vartheta \) of a INK-algebra \( \mathfrak{A} \) is order reversing, that is if \( \sigma \leq \tau \) then \( \vartheta (\sigma ) \geq \vartheta (\tau ) \) for all \( \sigma , \tau \in \mathfrak{A} \).

\textbf{Proof:}

Let \( \sigma , \tau \in \mathfrak{A} \) such that \( \sigma \leq \tau \)

Therefore \( \sigma \odot \tau = 0 \). Put \( \rho = 0 \).

Now, \( \vartheta (\sigma ) = \mu (\sigma \odot 0) \)

\[
\geq \min \{ \vartheta ((\sigma \odot \rho) \odot (\tau \odot \rho)), \vartheta (\tau ) \}
\]

\[
= \min \{ \vartheta ((\sigma \odot 0) \odot (\tau \odot \rho)), \vartheta (\tau ) \}
\]

\[
= \min \{ \vartheta (\sigma \odot \tau), \vartheta (\tau ) \}
\]

\[
\vartheta (\sigma ) = \vartheta (\tau )
\]

3.8 Theorem. A fuzzy set \( \vartheta \) in a INK-algebra \( \mathfrak{A} \) is a fuzzy \( P \)-ideal if and only if it is a fuzzy ideal of \( \mathfrak{A} \).

\textbf{Proof:} Let \( \vartheta \) be a fuzzy \( P \)-ideal of \( \mathfrak{A} \). Then

(i) \( \vartheta (0) \geq \vartheta (\sigma) \)

(ii) \( \vartheta (\sigma) \geq \min \{ \vartheta ((\sigma \odot \rho) \odot (\tau \odot \rho)), \vartheta (\tau ) \} \),

putting \( \rho = 0 \) in (ii) we have,

\[
\vartheta (\sigma) \geq \min \{ \vartheta (\sigma \odot \tau), \vartheta (\tau ) \}.
\]
Hence \( \mathcal{I} \) is a fuzzy-ideal of \( \mathfrak{F} \).

Conversely, \( \mathcal{I} \) is a fuzzy-ideal of \( \mathfrak{F} \).

\[
\mathcal{I}(\sigma) \geq \min \{ \mathcal{I}(\sigma \otimes \rho) \circ (\tau \otimes \rho), \mathcal{I}(\tau) \}
\]

\[
\mathcal{I}(\sigma) = \min \{ \mathcal{I}(\sigma \otimes \rho) \circ (\tau \otimes \rho), \mathcal{I}(\tau) \}
\]

which proves the result.

3.9 Theorem. Let \( \mathcal{I} \) be a fuzzy set in a INK-algebra \( \mathfrak{F} \) and let \( t \in \text{Im}(\mathcal{I}) \). Then \( \mathcal{I} \) is a fuzzy p-ideal of \( \mathfrak{F} \) if and only if the level subset: \( \mathcal{I}_t = \{ \sigma \in \mathfrak{F} \mid \mathcal{I}(\sigma) \geq t \} \) is a p-ideal of \( \mathfrak{F} \), which is called a level p-ideal of \( \mathcal{I} \).

Proof:

Assume that \( \mathcal{I} \) is a fuzzy p-ideal of \( \mathfrak{F} \). Clearly \( 0 \in \mathcal{I}_t \).

Let \((\sigma \otimes \rho) \circ (\tau \otimes \rho) \in \mathcal{I}_t \) and \( \tau \in \mathcal{I}_t \).

Then \( \mathcal{I}(\sigma \otimes \rho) \circ (\tau \otimes \rho) \geq t \) and \( \mathcal{I}(\tau) \geq t \).

Now \( \mathcal{I}(\sigma) \geq \min \{ \mathcal{I}(\sigma \otimes \rho) \circ (\tau \otimes \rho), \mathcal{I}(\tau) \} \)

\[
\geq \{ t, t \} = t
\]

Hence \( \mathcal{I}_t \) is p-ideal of \( \mathfrak{F} \).

Conversely, let \( \mathcal{I}_t \) is p-ideal of \( \Sigma \) for any \( t \in [0, 1] \).

Suppose assume that there exist some \( \sigma_0 \in \mathfrak{F} \) such that

\[
\mathcal{I}(0) < \mathcal{I}(\sigma_0)
\]

Take \( s = \frac{1}{2}[\mathcal{I}(0) + \mathcal{I}(\sigma_0)] \)

\[
\Rightarrow s < \mathcal{I}(\sigma_0) \text{ and } 0 \leq \mathcal{I}(0) \leq 1
\]

\[
\sigma_0 \in \mathcal{I}_t \text{, and } \mathcal{I} \notin \mathcal{I}_t \text{, a contradiction,}
\]

since \( \mathcal{I}_t \) is a p-ideal of \( \mathfrak{F} \).

Therefore, \( \mathcal{I}(0) \geq \mathcal{I}(\sigma) \).

If possible, assume that \( \sigma_0, \tau_0 \rho_0 \in \mathfrak{F} \)

\[
\mathcal{I}(\sigma_0) \geq \min \{ \mathcal{I}(\sigma_0 \otimes \rho_0) \circ (\tau_0 \otimes \rho_0), \mathcal{I}(\tau_0) \}:
\]

Take \( s = \frac{1}{2}[\mathcal{I}(\sigma_0) + \mathcal{I}(\sigma_0 \otimes \rho_0) \circ (\tau_0 \otimes \rho_0), \mathcal{I}(\tau_0)] \)

\[
\Rightarrow s > \mathcal{I}(\sigma_0) \text{ and } \mathcal{I} < \min \{ \mathcal{I}(\sigma_0 \otimes \rho_0) \circ (\tau_0 \otimes \rho_0), \mathcal{I}(\tau_0) \}
\]

\[
\Rightarrow s > \mathcal{I}(\sigma_0), s < \mathcal{I}(\sigma_0 \otimes \rho_0) \circ (\tau_0 \otimes \rho_0) \text{ and } s < \mathcal{I}(\tau_0)
\]

\[
\sigma_0 \notin \mathcal{I}_t \Rightarrow, \text{ a contradiction, since } \mathcal{I}_t \text{ is a p-ideal of } \Sigma.
\]

Therefore, \( \mathcal{I}(\sigma) \geq \min \{ \mathcal{I}(\sigma \otimes \rho) \circ (\tau \otimes \rho), \mathcal{I}(\tau) \}, \text{ for any } \sigma, \tau, \rho \in \mathfrak{F} \).

IV. CARTESIAN PRODUCT OF FUZZY P-IDEAL OF INK-ALGEBRAS

4.1 Definition. Let \( \mathcal{I} \) and \( q \) be the fuzzy set in a set \( \mathfrak{F} \). The Cartesian product \( \mathcal{I} \times q : \mathfrak{F} \times \mathfrak{F} \rightarrow [0, 1] \) is defined by,

\[
(\mathcal{I} \times q)(\sigma, \tau) = \min \{ \mathcal{I}(\sigma), q(\tau) \} \text{ for all } \sigma, \tau \in \mathfrak{F}.
\]

4.2 Theorem. If \( \mathcal{I} \) and \( q \) are fuzzy p-ideal in a INK-algebra \( \mathfrak{F} \), then \( \mathcal{I} \times q \) is a fuzzy p-ideal in \( \mathfrak{F} \times \mathfrak{F} \).
Proof:

For any $(\sigma, \tau) \in \mathfrak{F} \times \mathfrak{F}$, we have:

\[(\mathfrak{F} \times \mathfrak{F}) (0, 0) = \min \{ \mathfrak{F} (0), \mathfrak{F} (0) \} \geq \min \{ \mathfrak{F} (\sigma), \mathfrak{F} (\tau) \} = (\mathfrak{F} \times \mathfrak{F}) (\sigma, \tau).\]

Let $(\sigma_1, \sigma_2), (\tau_1, \tau_2)$ and $(\rho_1, \rho_2) \in \mathfrak{F} \times \mathfrak{F}$.

\[(\mathfrak{F} \times \mathfrak{F}) (\sigma_1, \sigma_2) = (\mathfrak{F} \times \mathfrak{F}) (\sigma_1 \odot \sigma_2) = \min \{ \mathfrak{F} (\sigma_1), \mathfrak{F} (\sigma_2) \} \geq \min \{ \min \{ \mathfrak{F} ((\sigma_1 \odot \rho_1) \odot (\tau_1 \odot \rho_1)), \mathfrak{F} (\tau_1) \}, \min \{ \mathfrak{F} ((\sigma_2 \odot \rho_2) \odot (\tau_2 \odot \rho_2)), \mathfrak{F} (\tau_2) \} \} = \min \{ \min \{ (\mathfrak{F} \times \mathfrak{F}) ((\sigma_1 \odot \rho_1) \odot (\tau_1 \odot \rho_1)), (\mathfrak{F} \times \mathfrak{F}) ((\sigma_2 \odot \rho_2) \odot (\tau_2 \odot \rho_2)), (\mathfrak{F} \times \mathfrak{F}) (\tau_1, \tau_2) \} \}

4.3 Theorem. Let $\mathfrak{F}$ and $\mathfrak{Q}$ be fuzzy set in a INK-algebra $\mathfrak{F}$ such that $\mathfrak{F} \times \mathfrak{F}$ is a fuzzy $p$-ideal of a INK-algebra in $\mathfrak{F} \times \mathfrak{F}$.

Then:

- (i) Either $\mathfrak{F} (0) \geq \mathfrak{F} (\sigma)$ or $\mathfrak{Q} (0) \geq \mathfrak{Q} (\sigma)$.
- (ii) If $\mathfrak{F} (0) \geq \mathfrak{F} (\sigma)$, then either $\mathfrak{Q} (0) \geq \mathfrak{Q} (\sigma)$ or $\mathfrak{Q} (0) \geq \mathfrak{Q} (\sigma)$.
- (iii) If $\mathfrak{Q} (0) \geq \mathfrak{Q} (\sigma)$, then either $\mathfrak{F} (0) \geq \mathfrak{F} (\sigma)$ or $\mathfrak{F} (0) \geq \mathfrak{F} (\sigma)$.

Either $\mathfrak{F}$ or $\mathfrak{Q}$ is a fuzzy $p$-ideal of $\mathfrak{F}$.

Proof:

$\mathfrak{F} \times \mathfrak{F}$ is a fuzzy $p$-ideal of $\mathfrak{F} \times \mathfrak{F}$. Therefore

\[(\mathfrak{F} \times \mathfrak{F}) (0, 0) \geq (\mathfrak{F} \times \mathfrak{F}) (\sigma, \tau),\]

and

\[(\mathfrak{F} \times \mathfrak{F}) (\sigma_1, \sigma_2) \geq \min \{ (\mathfrak{F} \times \mathfrak{F}) ((\sigma_1, \sigma_2) \odot (\rho_1, \rho_2)) \odot ((\tau_1, \tau_2) \odot (\rho_1, \rho_2)), (\mathfrak{F} \times \mathfrak{F}) (\tau_1, \tau_2) \} \]

Suppose that $\mathfrak{F} (0) < \mathfrak{F} (\sigma)$ and $\mathfrak{Q} (0) < \mathfrak{Q} (\tau)$.

\[(\mathfrak{F} \times \mathfrak{F}) (\sigma, \tau) = \min \{ \mathfrak{F} (\sigma), \mathfrak{Q} (\tau) \} \geq \min \{ \mathfrak{F} (0), \mathfrak{Q} (0) \} = (\mathfrak{F} \times \mathfrak{F}) (0, 0),\]

a contradiction.

Therefore, either $\mathfrak{F} (0) \geq \mathfrak{F} (\sigma)$ or $\mathfrak{Q} (0) \geq \mathfrak{Q} (\sigma)$ for all $\sigma \in \mathfrak{F}$.

Assume that there exists $\sigma, \tau \in \Sigma$ such that:

$\mathfrak{Q} (0) < \mathfrak{F} (\sigma)$ and $\mathfrak{Q} (0) < \mathfrak{Q} (\tau)$.

Then: $\mathfrak{F} \times \mathfrak{F} (0, 0) = \min \{ \mathfrak{F} (0), \mathfrak{Q} (0) \} = \mathfrak{Q} (0)$

$\mathfrak{F} \times \mathfrak{F} (\sigma, \tau) = \min \{ \mathfrak{F} (\sigma), \mathfrak{Q} (\tau) \} = (\mathfrak{F} \times \mathfrak{F}) (0, 0)$

a contradiction.

Hence if $\mathfrak{F} (0) \geq \mathfrak{F} (\sigma)$ for all $\sigma \in \mathfrak{F}$,
then either, \( q(0) \geq 9(\sigma) \) or \( q(0) \geq q(\sigma) \)

Similarly, we can prove that if \( q(0) \geq q(\sigma) \) for all \( \sigma \in \mathcal{F} \),
then either \( 9(0) \geq 9(\sigma) \) or \( 9(0) \geq q(\sigma) \).

First, we prove that \( q \) is a fuzzy p-ideal of \( \mathcal{F} \).
Since, by (i), either \( 9(0) \geq 9(\sigma) \) or \( q(0) \geq q(\sigma) \).
Assume that \( q(0) \geq q(\sigma) \).

It follows from (iii) that either \( 9(0) \geq 9(\sigma) \) or \( 9(0) \geq q(\sigma) \). If \( 9(0) \geq q(\sigma) \), then:

\[
q(\sigma) = \min\{9(0), q(\sigma)\} = (9 \times q)(0, \sigma).
\]

\[
q(\sigma) = \min\{9(0), q(\sigma)\} = (9 \times q)(0, \sigma) \geq \min\{(9 \times q)((0, \sigma) \ominus (0, \rho)) \ominus ((0 \ominus \tau) \ominus (0, \rho)),
(9 \times q)(0, \tau) = \min\{(9 \times q)\(((0 \ominus 0), (\sigma \ominus \rho)) \ominus ((0 \ominus 0), (\tau \ominus \rho))\),
(9 \times q)(0, \tau) = \min\{(9 \times q)((0 \ominus 0) \ominus (\sigma \ominus \rho) \ominus (\tau \ominus \rho)), (9 \times q)(0, \tau)
q(\sigma) = \min\{q((\sigma \ominus \rho) \ominus (\tau \ominus \rho)), q(\tau)\}
\]

Hence \( q \) is a fuzzy p-ideal of \( \mathcal{F} \).
Now we will prove that \( 9 \) is a fuzzy p-ideal of \( \Sigma \).
Let \( 9(0) \geq 9(\sigma) \). By (ii) either \( q(0) \geq q(\sigma) \) or \( q(0) \geq q(\sigma) \).
Assume that \( q(0) \geq q(\sigma) \).
Then:

\[
9(\sigma) = \min\{9(\sigma), q(0)\} = (9 \times q)(\sigma, 0).
\]

\[
9(\sigma) = \min\{9(\sigma), q(0)\} = (9 \times q)(\sigma, 0) \geq \min\{(9 \times q)\(((0, 0) \ominus (\sigma, 0)) \ominus ((\tau \ominus 0) \ominus (0, 0)),
(9 \times q)(\tau, 0) = \min\{(9 \times q)\(((0, 0) \ominus (\sigma \ominus \rho) \ominus (\tau \ominus \rho)), (9 \times q)(\tau, 0)
(9 \times q)(\tau, 0) = \min\{(9 \times q)\(((0 \ominus 0) \ominus (0 \ominus 0)), (9 \times q)(\tau, 0)
\]

Hence \( 9 \) is a fuzzy p-ideal of \( \mathcal{F} \).

V. HOMOMORPHISM OF INK-ALGEBRAS

5.1 Definition. Let \( \mathcal{F} \) and \( T \) be INK-algebras. A mapping \( f: \mathcal{F} \rightarrow T \) is said to be a homomorphism if it satisfies:

\[
f(\sigma \ominus \tau) = f(\sigma) \ominus f(\tau), \text{ for all } \sigma, \tau \in \mathcal{F}.
\]

5.2 Definition. Let \( f: \mathcal{F} \rightarrow \mathcal{F} \) be an endomorphism and \( 9 \) a fuzzy set in \( \mathcal{F} \). We define a new fuzzy in \( \mathcal{F} \) by \( 9_f \) in \( \mathcal{F} \) by \( 9_f(\sigma) = 9(f(\sigma)) \), for all \( \sigma \) in \( \mathcal{F} \).
5.3 Theorem. Let $f$ be an endomorphism of a INK-algebra $\mathfrak{I}$. If $\mathfrak{I}$ is a fuzzy p-ideal of $\mathfrak{I}$, then so is $\mathfrak{I}_f$.

Proof:

\[ \mathfrak{I}_f(\sigma) = \mathfrak{I} (f(\sigma)) \leq \mathfrak{I} (0). \] Let $\sigma, \tau, \rho \in \mathfrak{I}$. 

\[ \mathfrak{I}_f(\sigma) = \mathfrak{I} (f(\sigma)) \]

\[ \geq \min \{ \mathfrak{I} (f(\sigma) \circ f(\rho)) \circ (f(\tau) \circ f(\rho)), \mathfrak{I} (f(\tau)) \} \]

\[ = \min \{ \mathfrak{I} ((f(\sigma) \circ \rho) \circ f(\tau)), \mathfrak{I} (f(\tau)) \} \]

\[ = \min \{ \mathfrak{I}_f((\sigma \circ \rho) \circ (\tau \circ \rho)), \mathfrak{I}_f(\tau) \}. \]

Hence $\mathfrak{I}_f$ is a fuzzy p-ideal of $\mathfrak{I}$.

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