A Brief Overview of the Classical Transportation Problem

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ABSTRACT

The classical transportation problem (TP) is a distribution problem where commodities are transferred from many sources to many destinations with a least total cost. Also, TP considered as one of the classification of a linear programming (LP) problem, and it has a great alliance to inaugurate the linear program and its solution procedure. The variations of classical TP generally depend on the supply and demand constraints. The effectiveness of the algorithm for solving TP is determined by the closeness to the least cost solution to the TP. In this paper, the existence of solution to the TP, the basic theorems of classical TP, are stated and proven in a new manner. Also, an analysis has been performed to indicate the limitations of the existing solution procedures. Finally, the necessary and sufficient conditions are carried out for the optimality to the TP.

Keywords- Initial Feasible Solution, MODI, Optimal Solution, Transportation Problem, SSM, VAM
I. INTRODUCTION

The transportation problem (TP) transferring the commodities from several sources to several destinations to satisfy the demand constraints of the destinations with minimal total costs. To obtain optimal solution (OS) to the TP, the initial feasible solution (IFS) is needed for the existing solution procedures. The efficiency of the IFS is determined by the closeness of the IFS to the optimal solution in the existing solution procedure. The history of the transportation problem is lengthy, and as one of the variants of the linear programming (LP) problem, TP has a great impact to the establishment of the solution algorithm to the LP [1-3].

A hypothesis was developed for transporting soil during the forts and roads construction with minimal transport cost by Monge [4]. According to Schrijver [5], a number of solution approaches for cargo-transportation planning were proposed by Tolstoi [6-7]. Also, Tolstoi observed that the cycle condition is not only necessary but also sufficient (not given any proof) for optimality. Transportation problem was formulated and converted into a linear programming problem by Hitchcock in 1941. He used linear programming to minimize the total transport cost by transferring commodities from sources to destinations where the solution procedure was very close to the later established simplex method [8].

The Simplex method was developed by Dantzig for solving the LP problem efficiently in most of the cases [9]. Later, the Simplex method was applied by Dantzig to solve the TP and obtain the optimal solution which was published in 1951 [10]. Also, Dantzig proposed a new separate method to find the IFS to the TP [10] which was later named as North West Corner Rule (NWC) by Charnes and Cooper [11]. The first optimality test method called “Stepping Stone Method (SSM)” was developed by Charnes and Cooper [11]. Another optimality test method Modified Distribution Method (MODI) was developed in 1955. Reinfeld and Vogel developed the VAM algorithm to obtain IFS [12].

Later on, many researchers tried to improve the IFS algorithm. Sabbagh and Ghafari [13] developed a new algorithm to find IFS for a balanced TP where they claimed that their algorithm was five times faster than the Simplex Method and provided a degeneracy free solution. Ahmed et al. [14, 15] proposed a pair of algorithms, one of them is the Incessant Allocation Method, to attained better IFS to the TP. A greedy algorithm has been developed by converting the TP into a dual problem by Liu [16] to minimize the transport cost.

It is observed that researchers were modified the VAM algorithm to improve the initial feasible solution. Soomro et al. [17], Alkubaisi [18], Akpan et al. [19], Ullah et al. [20] are some of the examples of modified VAM algorithms. Recently the interval transportation problem (ITP) was studied by Cerulli et al. [21] and proposed an Iterative Local Search (ISL) algorithm to obtain a lower bound of the optimum solution. Also, Xie et al. [22] proposed a genetic algorithm for ITP and established the necessary and sufficient conditions to find the worst optimum value. Sharma et al. [23] obtained an IFS to the two-level hierarchical bottleneck (time minimization) TP by proposing a new polynomial algorithm. Also, Matsu and Scheifele [24] developed a linear time algorithm for the unbalanced Hitchcock transportation problem with runtime $O(m(n!)^2)$.

The combination of VAM and MODI algorithms are most popular to obtain the optimal solution to the TP since 1958. There are many IFS algorithms available in the literature; some of which are new TP algorithms [25-29] and most of them are modification to the earlier TP algorithms [30-33]. But, it has been observed that no TP algorithm could guarantee the optimal solution to the TP without based on IFS.

The theoretical development of such an optimal solution algorithm is important. As a class of linear programming (LP) problem, transportation problem has always been a feasible solution, and there exists at least one optimal solution. However, the existing feasibility condition does not lead to the optimality condition. Also, the other existing properties of the solution to the TP is not quite sound. In this paper, the basic theorems of the classical TP are stated and proven. Also, the limitations of the existing optimal solution procedures to the TP are analyzed, and the reasons for the existing transportation algorithms not being the optimal algorithm are revealed as well.

In Section 2, modifications to the existing theorems of TP are proposed along with sound proofs. All the current solution techniques to the TP are discussed in section 3. In Section 4, the limitations of the current solution techniques to the TP are analyzed. Section 5 explains some of the reasons for which the existing transportation algorithms are not optimal algorithm. The scopes of the further research to improve the solution techniques to the TP are discussed in section 6. In section 7, some criteria how to improve the solution techniques to the TP are proposed.
II. MATERIALS AND METHODS

Existence of Solution to the Transportation Problem

The existing feasibility condition to the TP found in the literature is \( \sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j \), where \( s_i \) be the supply amount for the source \( i \) and \( d_j \) be the demand for the destination \( j \); and \( m \) denotes the total number of sources and \( n \) denotes the total number of destinations. But it is known that the unbalanced TP is also feasible when \( \sum_{i=1}^{m} s_i > \sum_{j=1}^{n} d_j \). Hence, the existing feasibility condition to the TP is not sufficient to prove that TP is always feasible. In this study, the new feasibility condition is introduced to the TP to prove that TP is always feasible. Also, the upper bound is set for the basic variables in the feasible solution (FS) to the unbalanced TP. The following theorems show that the TP has both the FS and OS. Also, the modified feasibility condition leads to the necessary and sufficient (N-S) condition for optimal solution to the TP.

Theorem 1. (Existence of a Feasible Solution)

The solution to the transportation problem is feasible if and only if

\[
\sum_{j=1}^{n} \sum_{i=1}^{m} x_{ij} = \sum_{j=1}^{n} d_j
\]

where \( x_{ij} \) is the allocation to the cell \((i, j)\) and \( d_j \) be the demand of \( j^{th} \) destination.

Proof. The existing N-S condition for the existence of a FS to the TP is that the total volume of commodity required at different destinations must be satisfied by the amount of the total commodity available at the different sources. But the whole supply quantity and the whole demand quantity may or may not be identical. Therefore, two cases arises as follows: (i) \( \sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j \) and (ii) \( \sum_{i=1}^{m} s_i \neq \sum_{j=1}^{n} d_j \), where \( s_i \) be the supply quantity of \( i^{th} \) source and \( d_j \) be the demand quantity of \( j^{th} \) destination.

For the case (ii) when \( \sum_{i=1}^{m} s_i \neq \sum_{j=1}^{n} d_j \), the total supply quantity must be larger than the total demand quantity, i.e., \( \sum_{i=1}^{m} s_i > \sum_{j=1}^{n} d_j \) in the classical TP because of when \( \sum_{i=1}^{m} s_i < \sum_{j=1}^{n} d_j \) then \( \sum_{j=1}^{n} \sum_{i=1}^{m} x_{ij} \neq \sum_{j=1}^{n} d_j \) which contradicts the general demand constraints of TP.

Therefore, for both the cases, it is enough to prove that the N-S condition for the existence of a FS to the classical TP is

\[
\sum_{j=1}^{n} \sum_{i=1}^{m} x_{ij} = \sum_{j=1}^{n} d_j
\]  

(1)

Necessary Condition: Let us consider the solution to the TP is feasible. For case (i), we get \( \sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j \). We consider the mechanism of the distribution of supply amount from \( i^{th} \) sources to \( j^{th} \) destinations strictly maintain a ratio or proportionality to optimize the objective function of transportation problem.

Let \( \rho_{ij} \) be the ratio or proportionality factor for the allocation from \( i^{th} \) sources to \( j^{th} \) destinations which satisfies the following conditions:

\[
\sum_{i=1}^{m} \rho_{ij} = \sum_{j=1}^{n} \rho_{ij} = 1; \quad \forall \, i = 1, 2, \ldots, m; \quad j = 1, 2, \ldots, n;
\]

(2)

Therefore the allocation from \( i^{th} \) sources to \( j^{th} \) destinations be

\[
x_{ij} = \rho_{ij}d_j; \quad \forall \, i, j
\]

(3)

where \( x_{ij} \geq 0 \) since \( s_i > 0 \) and \( d_j > 0 \).

Now form (3),
\[
\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} = \sum_{j=1}^{n} \sum_{i=1}^{m} \rho_{ij} d_j
\]

\[
\Rightarrow \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} = \sum_{j=1}^{n} d_j \sum_{i=1}^{m} \rho_{ij}
\]

\[
\Rightarrow \sum_{j=1}^{n} \sum_{i=1}^{m} x_{ij} = \sum_{j=1}^{n} d_j; \quad \text{[using (2)]}
\]

which satisfies the necessary condition.

**Sufficient Condition:** For the case (i), let the transportation problem satisfies all the supply and demand constraints. That is,

\[
\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} = \sum_{i=1}^{m} s_i
\]

and

\[
\sum_{j=1}^{n} \sum_{i=1}^{m} x_{ij} = \sum_{j=1}^{n} d_j
\]

Therefore, we get,

\[
\sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j
\]

which proves the sufficient condition.

For the case (ii), when the entire supply quantity and the entire demand quantity are not identical, we get:

\[
\sum_{i=1}^{m} s_i > \sum_{j=1}^{n} d_j \quad (4)
\]

Here, the total supply amount is sufficient to distribute the commodity to satisfy the demand of destinations. The additional supply amount are kept as unused in the inventory.

Let the additional supply amount be,

\[
w = \sum_{i=1}^{m} s_i - \sum_{j=1}^{n} d_j
\]

and let \(s_{i}^{*}\) be the reduced \(i^{th}\) supply capacity where

\[
\sum_{i=1}^{m} s_{i}^{*} = \sum_{i=1}^{m} s_i - w
\]

Therefore (4) can be rewritten as

\[
\sum_{i=1}^{m} s_{i}^{*} = \sum_{j=1}^{n} d_j
\]

which follows the case (i).

Therefore, in both case (i) and (ii), the transportation problem always has a feasible solution.
Theorem 2. (Upper Bound to the Number of Basic Variables)

Assume that TP is feasible. If \( \sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j \) then TP has exactly \((m + n - 1)\) basic variables and if \( \sum_{i=1}^{m} s_i \neq \sum_{j=1}^{n} d_j \) then number of basic variables bounded by \((m + n - 2)\).

Proof. First Part: For the given condition \( \sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j \), we can write the supply and demand constraints be

\[
\sum_{j=1}^{n} x_{ij} = s_i; \quad \text{for } i = 1,2,3, \ldots m \tag{5}
\]

\[
\sum_{i=1}^{m} x_{ij} = d_j; \quad \text{for } j = 1,2,3, \ldots n \tag{6}
\]

Also, the total allocation from all sources to all destinations is equal to the total supply amount. That is:

\[
\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} = \sum_{i=1}^{m} s_i \tag{7}
\]

Similarly, the total allocation is equal to the total amount of demand. That is:

\[
\sum_{j=1}^{n} \sum_{i=1}^{m} x_{ij} = \sum_{j=1}^{n} d_j \tag{8}
\]

Now we adding the first \((n - 1)\) constraints in (8),

\[
\sum_{j=1}^{n-1} \sum_{i=1}^{m} x_{ij} = \sum_{j=1}^{n-1} d_j \tag{9}
\]

Subtracting (9) from (8), we get,

\[
\sum_{i=1}^{m} x_{in} = d_n \tag{10}
\]

which is the \(n^{th}\) demand constraint in (6).

Similarly, we can get,

\[
\sum_{j=1}^{n} x_{mj} = s_m \tag{11}
\]

which is the \(m^{th}\) supply constraint in (5). Therefore, any one of the \((m + n)\) constraints can be derived from the remaining \((m + n - 1)\) constraints. So there are \((m + n - 1)\) independent constraints which lead to the \((m + n - 1)\) basic variables in the feasible solution to the TP.

Second Part: For the given condition \( \sum_{i=1}^{m} s_i \neq \sum_{j=1}^{n} d_j \), two cases may arise; which are (i) \( \sum_{i=1}^{m} s_i > \sum_{j=1}^{n} d_j \) and (ii) \( \sum_{i=1}^{m} s_i < \sum_{j=1}^{n} d_j \).

Case-1: For \( \sum_{i=1}^{m} s_i > \sum_{j=1}^{n} d_j \), we can write for all supply and demand constraints,

\[
\sum_{j=1}^{n} x_{ij} \leq s_i; \quad \text{for } i = 1,2,3, \ldots m \tag{12}
\]
\[ \sum_{i=1}^{m} x_{ij} = d_j; \quad \text{for } j = 1, 2, 3, \ldots n \] (13)

Therefore, together with all \( m \) supply and \( n \) demand constraints, we get,
\[ \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} \leq \sum_{i=1}^{m} s_i \] (14)
\[ \sum_{j=1}^{n} \sum_{i=1}^{m} x_{ij} = \sum_{j=1}^{n} d_j \] (15)

Let us consider,
\[ w_i = s_i - \sum_{j=1}^{n} x_{ij} \geq 0; \quad \text{for } i = 1, 2, 3, \ldots m \] (16)
and,
\[ \sum_{i=1}^{m} w_i > 0 \]

Now, we can rewrite (14) as follows:
\[ \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} = \sum_{i=1}^{m} s_i - \sum_{i=1}^{m} w_i \] (17)

Now, adding the first \((m - 1)\) constraints in (17),
\[ \sum_{i=1}^{m-1} \sum_{j=1}^{n} x_{ij} = \sum_{i=1}^{m-1} s_i - \sum_{i=1}^{m-1} w_i \] (3.26)

Subtracting (18) from (17), we get,
\[ \sum_{j=1}^{n} x_{mj} = s_m - w_m \]
\[ \Rightarrow \sum_{j=1}^{n} x_{mj} \leq s_m \quad [\because w_m \geq 0] \]

which is the \( m^{th} \) supply constraint in (12). Therefore, it is possible to derive any one of the \( m \) supply constraints from the remaining \((m - 1)\) independent supply constraints. Similarly, it can easily be shown that any one of the \( n \) demand constraints can derive from the remaining \((n - 1)\) independent demand constraints. Therefore, there are total of \((m + n - 2)\) independent supply and demand constraints.

Similarly, for Case-2, we can show that the number of independent constraints are \((m + n - 2)\).

So, it can be concluded that there are exactly \((m + n - 1)\) basic variables involves in the feasible solution to the balanced problem and the upper bound to the total of basic variables in the FS to the unbalanced TP are bounded by \((m + n - 2)\).

Theorem 3. (Existence of the Optimal Solution)

Assume that TP is feasible. Then, TP has at least one optimal solution.

Proof. Theorem 1 shows that TP is feasible for \( \sum_{i=1}^{m} s_i \geq \sum_{j=1}^{n} d_j \). It follows that each of the basic variable \( x_{ij} \) is bounded by \( 0 \leq x_{ij} \leq \min\{s_i, d_j\}; \quad \forall \ i, j \).
Also Theorem 2 shows that the total non-negative basic variables in the FS to the TP is exactly number of \((m + n - 1)\) or bounded by \((m + n - 2)\).

Thus the feasible region of the problem is closed, bounded and non-empty and hence there exists at least one OS to the TP.

The existing feasibility condition i.e., \(\sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j\) to the TP does not reflect the optimality condition; since TP has a feasible solution not only for the equality constraints but also for inequality constraints of supply and demand that is \(\sum_{i=1}^{m} s_i > \sum_{j=1}^{n} d_j\). Also, the optimal solution does not depend on whether the TP is balanced or unbalanced, rather it depends whether the total shipment satisfies the total demand. The following theorem states that the N-S conditions for the OS to the TP.

**Theorem 4. (Necessary and Sufficient Condition for Optimality)**

Consider that TP is feasible and therefore

\[
\sum_{j=1}^{n} \sum_{i=1}^{m} x_{ij} = \sum_{j=1}^{n} d_j
\]

A FS to the TP is optimal if and only if the basic variables \(x_{ij} \forall i, j\) are optimal.

**Proof.** Necessary Condition: Consider a FS which is optimal. Therefore, the demand of each of the destinations is satisfied by sum of the total shipment in that column. That is:

\[
\sum_{i=1}^{m} x_{ij} = \sum_{j=1}^{n} d_j ; \forall j = 1, 2, ..., n
\]

And also, the total transportation cost that is \(\sum_{j=1}^{n} \sum_{i=1}^{m} x_{ij} c_{ij}\) of the TP is minimal. Thus, all the destinations fulfill its demand and the total transport cost is minimal and hence each of the non-negative basic variables \(x_{ij}\) is optimal.

Sufficient Condition: Consider that the basic variable \(x_{ij}\) is optimal. Therefore, the sum of the allocation in each column must satisfy the demand of the destinations. That is:

\[
\sum_{i=1}^{m} x_{ij} = \sum_{j=1}^{n} d_j ; \forall j = 1, 2, ..., n
\]

And also, for each of the optimal \(x_{ij}\) the total shipment is optimal which lead to the lowest transport cost.

**Existing Solution Procedure to the Transportation Problem**

The OS to the TP in the classical transportation model depends on the initial feasible solution (IFS). There are a huge number of algorithms available in the literature to obtain IFS to the TP which were discussed in Chapter-1 where VAM is known as efficient algorithm to obtain IFS. And, the MODI and Steeping Stone Method (SSM) are the only two OS algorithms to obtain the optimal solution to the TP based on the IFS.

**Basic Steps of Solution Procedure to the Classical Transportation Model**

The basic steps to obtain the OS to the classical TP are:

Step-1: Mathematical model of the TP.

Step-2: Determining whether the given problem is balanced or unbalanced (total supply is greater than total demand). If not than make it balanced by adding dummy columns with zero transport costs.

Step-3: Determining the IFS to the TP.

Step-4: Checking whether the IFS obtained in Step-3 is optimal. If not, obtain the optimal solution using MODI or SSM algorithm.
Initial Feasible Solution (IFS)

The VAM is widely used and considered as effective algorithm to obtain IFS to the TP. Some of the transportation algorithms were developed to improve or modify the VAM algorithm, which provide better IFS than VAM in some cases of transportation problem. Sometimes the IFS algorithms provide optimal solution but the percentage of this phenomenon is very low. There is no such an algorithm in the literature which provides optimal solution directly instead of IFS to the transportation problems.

Optimal Solution

There always exists an OS to the transportation problem. The optimal solution is determined from the IFS to the TP. The optimal solution algorithms can, of course, be performed only if

a. The number of allocations are \((m + n - 1)\).

b. These \((m + n - 1)\) allocations should be placed in such a position so that each of the rows and columns has at least one allocation.

There exist only two algorithms in the literature to obtain OS to the classical TP which are known as the SSM and the MODI. The optimal solution algorithms involve examination of each of the unallocated cells to observe is there any new allocation is possible or not to decrease the total transport cost.

Basic Steps of Solution Procedure to the Linear Programming Model of Transportation Problem

The underlying steps to obtain the optimal solution to the LP Model of the TP are given below:

Step-1: Formulate the transportation matrix of the TP.

Step-2: Convert this transportation matrix into the linear programming problem.

Step-3: Convert this general LP problem into a standard form of LP.

Step-4: Using any appropriate linear programming algorithm like Simplex Algorithm, Big-M Method etc. to obtain optimal solution.

III. RESULTS AND DISCUSSION

Limitations of the Existing Solution Procedure

From the above discussions of the existing solution procedures there is no such transportation algorithm in the literature which provides the optimal solution directly without based on IFS to the transportation problem. Even though the simplex type algorithm can provide optimal solution directly to the TP when TP is converted to the linear programming problem. But, these simplex type algorithms are not transportation algorithm. Here we identify some of the limitations of the existing solution procedure to the TP.

a) Linear Programming Techniques are not efficient for solving transportation problem

Since TP is one of the classification of a LP problem. The simplex algorithm and the other simplex type algorithms are efficient for solving LP. However, the linear programming techniques are not efficient for solving TP as it is not possible to apply simplex algorithm directly to TP until the TP is converted into LP. Also, there are too many variables involve in the TP when it is converted into LPP. For the size of \((m \times n)\) TP, the corresponding LP contains \(mn\) basic variables and \((m + n)\) slack and surplus variables with \((m + n)\) artificial variables, that is total of \(mn + 2(m + n)\) variables of \((m + n) \times (mn + 2(m + n))\) order matrix. Therefore, when the size of the TP increases the corresponding LP is not praiseworthy to obtain the optimal solution.

b) Two different types of methods are barriers in obtaining the optimal solution to the classical TP model

The classical TP model needs two different types of algorithms to attain an optimal solution; one for finding IFS and another for testing and finding the optimal solution. It is very rare that the IFS to the TPs are optimal especially when the problem size increases. Also, there are huge numbers of transportation algorithms available in the literature to obtain IFS from which is very tough to choose the appropriate algorithm for the different types and sizes of TPs. There are two optimality test methods available in the literature for testing and finding the OS to the TP such as MODI and SSM. But, these optimality test methods cannot obtain the OS without
based on the IFS. Even in degeneracy case, that is, when the total of basic variables is fewer than \((m + n - 1)\) then the optimal methods cannot apply to the obtained IFS. In that case, every TP algorithm in the literature need to make it balance when the TP is unbalanced (by adding dummy column when total supply quantity is larger than total demand quantity) to obtain the optimal solution. Also, it is not possible to determine that whether an IFS is optimal without applying the optimality test methods to the IFS.

c) **Optimality test method fails in some cases**

Sometime optimality test method cannot apply to the IFS even in non-degenerate case. It was shown by [34] that when an allocation is alone both in its associated row and column in a special case of allocating zero supply and zero demand, then the optimality test method cannot be applied to such an IFS. That is, MODI or SSM needs exactly \((m + n - 1)\) number of basic variables, and each of these allocations needs at least one more allocation in their associated row or column in the IFS to obtain the OS. To fulfill the above condition, no transportation algorithm (even the VAM) is available in the literature.

d) **The classical transportation algorithm is not efficient to solve some of the variants of Transportation Problem**

Some of the variants of TP like Interval TP, Multi-Objective TP, Fixed Charge TP needs meta-heuristic approach to obtain the optimal solution. In those cases, these variants depend on linear programming approach most of the time. For example, there are huge number of particular TP involves in a single interval transportation problem. Also, two types of algorithms (IFS and MODI/SSM) to obtain the optimal solution are not the efficient way to solve these variants of TP. Therefore, the optimal solution algorithm shown here may reduce the computational steps and time to solve these variants of TP.

**Factors behind the Existing Transportation Algorithms to make them non-optimal**

The following factors make the existing transportation algorithms are non-optimal.

a) **Crossed out row/column when satisfied the supply or demand constraint**

Almost in each of the transportation algorithms, there is a common operational step that is crossed out or blocked the row/column when it is satisfied by an allocation. This is the main reason for the initial feasible solution (IFS) not to be an optimal solution obtained by a transportation algorithm. When any row/column is crossed out or blocked by satisfying the corresponding row/column, this row/column will not be considered for further calculation. That is why, if this allocation is not optimal then the final IFS is not optimal. Unfortunately, there is no such IFS algorithm which selects the cost cells for allocations and guarantees the optimality. Therefore, this is the first barrier on the way of making a transportation algorithm optimal.

It has been observed in both the MODI and the SSM algorithms that no row/column is crossed out or blocked when introducing a new allocation. In each of the iterations, a new allocation is introduced, for which, any of the old allocations must leave and this iterative process is continued until achieving the optimal solution. It is also observed in the simplex algorithm that no row/column is crossed out or blocked when a new basis is introduced, for which another existing basis must leave. This is one of the criterion for which the MODI/SSM and the simplex type algorithm lead to the optimal solution where the transportation algorithms fail.

b) **Cannot change the Basic variables**

When a unit transport cost is selected for an allocation, the minimum amount of supply and demand associated to that unit transport cost is transported to the destination to fulfill or partially fulfill its demand. The rest of the supply or the demand is adjusted later. The variable \(x_{ij}\) used to denote that transported amount of commodity is known as the basic variable. The values of the basic variables determined this way are fixed after their allocation and cannot be changed in the whole processes of allocation of resources. There is no guarantee that the obtained basic variables will always be optimal. Hence, the solution obtained to a transportation problem by following the classical TP algorithms may not be optimal.

It has been observed that only the basic variables, in the MODI or SSM algorithm, are changed in each of the iterations. Because in each of the iterations, a new basic variable is introduced replacing an old one. Also in the simplex type algorithms, current basis is frequently changed by the newly introduced basis. So, there is always a scope to replace the non-optimal basic variables or basis by the optimal basic variables or basis. That is why, these algorithms always lead to the optimal solution.
c) Cycle criterion is absent in classical TP algorithms

It was reported by Tolstoi in 1920 that the cycle criterion is necessary to obtain the OS to the TP. But, it has been observed in this study that in all the existing transportation algorithms there is no cycle criterion to make it an optimal algorithm. In the classical transportation algorithm, each of the iterations selects a new basic variable and crossed out this associate row/column when the adjustment is completed. Therefore, this algorithm is terminated by force when all the rows and columns are crossed out or blocked. But in MODI and SSM algorithm along with the simplex type algorithm the calculations are repeated for a certain amount of time until it reach to the optimal solution.

Scope of Further Research for an Improved Solution Procedure to the TP

Based on the above discussion on limitations of the existing solution procedure to the TP, there lies a scope for improvement to the solution techniques to TP. The following steps of improvement of the solution techniques to the TP can be employed.

Development of an optimal transportation algorithm without based on an IFS

There is no transportation algorithm available in the literature which provides the optimal solution without based on IFS. The optimal solution algorithm reduces the hassle of using two types of algorithms in obtaining the OS to the TP. Also, the OS algorithm may reduce the computational time more effectively specifically for the large scale problems.

Improvement of the optimal solution algorithm into a polynomial time algorithm

The polynomial time algorithm is where the number of computational steps is the polynomial function of the size of the problem. For example, in most of the transportation algorithm the number of steps to obtain IFS is exactly $(m + n - 1)$, but in MODI and SSM method, the number of computational steps are not bounded. Therefore, there lies a scope for the improvement of such an optimal solution algorithm to a polynomial time algorithm. There also lies a further opportunity of development to the strongly polynomial time algorithm (optimal solution with exactly $(m + n - 1)$ iterations).

Optimality Condition for Transportation Problem

From the above discussion of the limitations of existing solution procedures and the reasons behind them, we set three key criteria to improve the transportation algorithm to the optimal solution algorithm.

Selection Criteria for making allocation

The number of basic variables in the solution to the TP is exactly $(m + n - 1)$ for balanced problem and bounded by $(m + n - 2)$ for the unbalanced problem. The first condition for the optimal cost solution to the TP is to improve the selection criteria of basic variables in each computational step. If it is possible to select the basic variables in computational steps, which belongs to the set of optimal $(m + n - 1)$ number of basic variables or very close to that one, the solution leads to the optimal solution or very close to the optimal solution.

Change of Basic variables

There should be a scope of change of a basic variable in a transportation algorithm, that is, when a new basic variable is introduced in a computational step then the new basic variable may or may not replace any current basic variable. But no row/column should be crossed out or blocked after introducing a new basic variable.

Cycle Criterion

This is a general criterion for optimality which is absent in the transportation algorithms. Generally, the computational steps in an optimal solution algorithm repeated for a certain amount of time based on the optimality condition.
IV. CONCLUSION

In this paper, our main focus is to discuss the basic theorems to the classical transportation problem so that the optimality conditions of the solution to the TP are satisfied, where the proofs of these theorems are clear and sound. We analyze the limitations of the existing solution procedures to the TP, which shows the necessity of developing the optimal solution algorithm without based on IFS. Also, we analyze some reasons for why the existing transportation algorithms are not to be the optimal algorithm. At the same time, we proposed three criteria to overcome the limitations and resolved the reasons and develop or improve the optimal solution algorithm which will follow the polynomial or strong polynomial time algorithm of TP.

V. FUTURE SCOPE

According to the above discussion, it is necessarily important to develop an optimal solution algorithm without based on IFS to the TP. Moreover, it will help to solve the variants of the TP in further studies.

Conflict of Interest: This original copy has not been distributed and isn't getting looked at for publication somewhere else. We have no irreconcilable situations to unveil.

REFERENCES


